Name : ______________________

HURLSTONE AGRICULTURAL HIGH SCHOOL
YEAR 12 2011 MATHEMATICS
ASSESSMENT TASK 1

Examiners: P. Biczo, S. Gee

General Instructions

- Reading time : 3 minutes
- **Working time : 40 minutes**
- Attempt **all** questions
- **Start a new sheet of paper for each question**
- All necessary working should be shown
- This paper contains 4 questions worth 8 marks each. Total Marks: **32 marks**
- Marks may not be awarded for careless or badly arranged work
- Board approved calculators may be used
- This examination paper must **not** be removed from the examination room

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Question 1 (Start a new sheet of paper) **Marks**

(a) Evaluate

(i) $\lim_{x\to\infty} \dfrac{x^2-3x-1}{2x^2-1}$ **1**

(ii) $\lim_{x\to 0} \dfrac{x}{x^2-1}$ **1**

(iii) $\lim_{x\to 1} \dfrac{x-1}{x^2-1}$ **1**

(b) Use the definition of the derivative by first principles,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h},$$

to show that if $f(x)=x^2-5x$, then $f'(x)=2x-5$ **3**

(iii) Draw a sketch of a possible curve that is differentiable in the domain $-a\le x\le a.$ **2**

Question 2 (Start a new sheet of paper) **Marks**

(a) Differentiate, with respect to x:

(i) $x^4 - 3x^2 + 5$ **1**

(ii) $\left(2x^2 + 5\right)^4$ **1**

(iii) $\dfrac{1}{\sqrt{x}}$ **1**

(iv) $\left(x^2 + 1\right)\left(3x - 1\right)^2$ **using the product rule** **2**

(v) $\dfrac{3x + 5}{4 - 5x}$ **2**

(b) Write down a version of the chain rule for finding the derivative of a function. **1**

Question 3 (Start a new sheet of paper)

(a) Consider the curve $f\left(x\right) = x^3 - 6x^2 + 9x + 1$.

(i) Find the coordinates of any stationary points and determine their nature. **3**

(ii) Sketch the curve, showing this information. **1**

(iii) Find any points of inflexion. **2**

(b) By using calculus, show that the curve $y = \dfrac{3}{x}$ is decreasing for all values of x for which it is defined. Justify your answer. **2**

Question 4 (Start a new sheet of paper) **Marks**

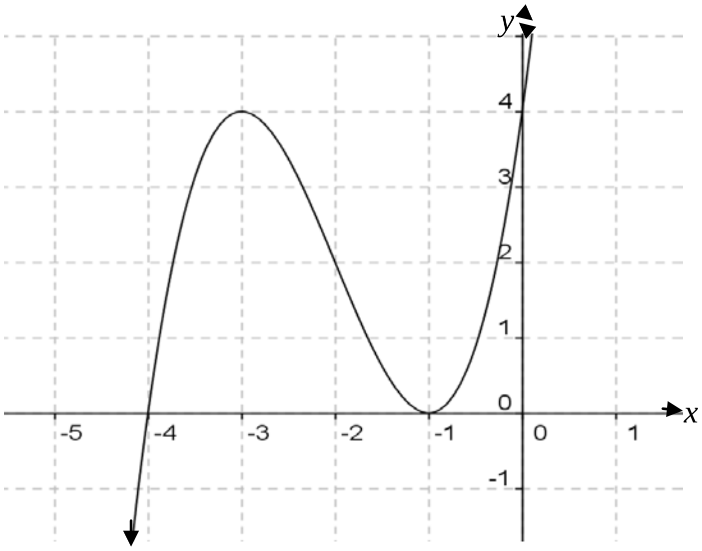
(a) Consider the curve $y = x^2 - 4x + 1$.

(i) Show that the gradient of the tangent at $x = 4$ is 4. **1**

(ii) Find the equation of the normal to the curve at the point where $x = 4$. **2**

(iii) At what point on the curve is the angle of inclination of the tangent $45°$? **2**

(b) The graph shown is $y = f(x)$. It is given that the only value for x at which $f''(x) = 0$ is when $x = -2$.



(i) For what values of x is $y = f(x)$ increasing? **1**

(ii) For what values of x is $f''(x) < 0$? **1**

(iii) Is the point $(-2, 2)$ a horizontal point of inflexion? **Justify** your answer. **1**

Year 12	Mathematics	HSC Task 1 2010/2011
Question No.1 & 2 Guidelines	Solutions and Marking	

Outcomes Addressed in this Question

P6 relates the derivative of a function to the slope of its graph.
P7 determines the derivative of a function through routine application of the rules of differentiation
P8 understands and uses the language and notation of calculus.

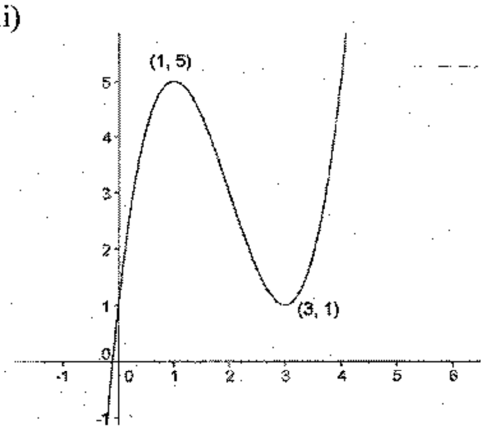
Outcome	Solutions	Marking Guidelines
	Question 1	
	a	
P8	*i)* $\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{3x}{x^2}-\frac{2}{x^2}}{\frac{2x^2}{x^2}-\frac{1}{x^2}}$ $=\lim_{x\to\infty}\frac{1-\frac{3}{x}-\frac{2}{x^2}}{2-\frac{1}{x^2}}$ $=\frac{1}{2}$	1 mark method leading to correct answer
	ii) $\lim_{x\to 0}\frac{x}{x^2-1}$ $=\frac{0}{0-1}=0$	1 mark method leading to correct answer
	iii) $\lim_{x\to 1}\frac{x-1}{x^2-1}$ $=\lim_{x\to 1}\frac{x-1}{(x-1)(x+1)}$ $=\lim_{x\to 1}\frac{1}{x+1}=\frac{1}{2}$	1 mark method leading to correct answer
	b	
P6	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f(x)=x^2-5x$ $f(x+h)=(x+h)^2-5(x+h)=x^2+2xh+h^2-5x-5h$ $f'(x)=\lim_{h\to 0}\frac{x^2+2xh+h^2-5x-5h-(x^2-5x)}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2-5h}{h}$ $=\lim_{h\to 0}(2x+h-5)$ $=2x-5$	3 marks correct method leading to correct conclusion 2 marks substantially correct solution 1 mark elementary progress towards correct solution
	c	
P5, 6	Any continuous smooth curve with end points at $x=-a$ and $x=a$.	1 mark suitable curve 1 mark clearly indicated end points.

	Question 2 **a** *i)* $\frac{d}{dx}\left(x^4-3x^2+5\right)=4x^3-6x$	1 mark correct answer
P7	*ii)* $\frac{d}{dx}\left[\left(2x^2+5\right)^4\right]=4\left(2x^2+5\right)^3\times 4x$ $=16\left(2x^2+5\right)^3$	1 mark correct answer
	iii) $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)=\frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$ $=-\frac{1}{2}x^{-\frac{3}{2}}$ $=\frac{-1}{2\sqrt{x^3}}$	1 mark correct answer
	iv) Let $y=\left(x^2+1\right)(3x-1)^2$ $u=x^2+1 \quad u'=2x$ $v=(3x-1)^2 \quad v'=6(3x-1)$ $\frac{d}{dx}(uv)=vu'+uv'$ $=(3x-1)^2\times 2x+\left(x^2+1\right)\times 6(3x-1)$ $=2(3x-1)\left(x(3x-1)+3\left(x^2+1\right)\right)$ $=2(3x-1)\left(6x^2-x+3\right)$	2 marks correct method leading to correct answer 1 mark substantially correct solution
	v) $\frac{d}{dx}\left(\frac{3x+5}{4-5x}\right)$ $u=3x+5 \quad v=4-5x$ $u'=3 \quad v'=-5$ $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$ $\frac{d}{dx}\left(\frac{3x+5}{4-5x}\right)=\frac{3(4-5x)--5(3x+5)}{(4-5x)^2}$ $=\frac{37}{(4-5x)^2}$	2 marks correct method leading to correct answer 1 mark substantially correct solution
	b $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$	1 mark correct answer

Outcomes Addressed in this Question

H6 Uses the derivative to determine features of the graph of a function

H5 applies appropriate techniques from the study of calculus and geometry to solve problems

Outcome	Solutions	Marking Guidelines
H6	a)(i) $y = x^3 - 6x^2 + 9x + 1$ $y' = 3x^2 - 12x + 9 = 0$ for stationary points $\therefore 3(x-3)(x-1) = 0$ $\therefore$ stationary points at $x = 3$ and $x = 1$ $y'' = 6x - 12$ When $x = 1$, $y'' = -6 < 0$ $\therefore$ concave down & so relative maximum at $x = 1$ When $x = 3$, $y'' = 6 > 0$ $\therefore$ concave up & so relative minimum at $x = 3$ Maximum at $(1,5)$, Minimum at $(3,1)$	3 marks : puts $y' = 0$ and correctly solves; finds y values at stationary points; and correctly justifies and determines nature 2 marks: substantial progress towards correct solution 1 mark: significant progress towards correct solution
H6	(ii) 	1 mark: correct graph or equivalent
H6	(iii) Possible points of inflexion at $y'' = 0$ $\therefore 6x - 12 = 0$ $\therefore x = 2$ is a possible point of inflexion. x: 1, 2, 3 y'': -6, 0, $+6$ Since the concavity changes, $x = 2$ is a point of inflexion	2 mark : finds pt.of inflexion & justifies concavity change 1 mark : one of above
H5	(b) $y = \dfrac{3}{x} = 3x^{-1}$ $\dfrac{dy}{dx} = -3x^{-2} = \dfrac{-3}{x^2}$ Since x^2 is positive for all values of x (except $x = 0$ where it is undefined), $\dfrac{dy}{dx}$ is negative for all $x \neq 0$. $\therefore y$ is decreasing.	2 mark : correct derivative and justification 1 mark : one of above

Year 11 Mathematics task 1 HSC 2010

Question No. 4 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5-applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems
P6-relates the derivative of a function to the slope of its graph
H6-uses the derivative to determine the features of the graph of a function
P8-understands and uses the language and notation of calculus
H7-uses the features of a graph to deduce information about the derivative

Outcome	Solutions	Marking Guidelines
	4.	
P6,P8	**a)i)** $y = x^2 - 4x + 1$ $\frac{dy}{dx} = 2x - 4$ at $x = 4$ $\frac{dy}{dx} = 2(4) - 4 = 4$ $\therefore$ gradient of tangent at $x = 4$ is 4.	**1 mark for complete correct solution**
H5	**ii)** Gradient of tangent is $m_1 = 4$ Gradient of normal is $m_1 = -\frac{1}{4}$ **Equation of normal is:** $y - 1 = -\frac{1}{4}(x - 4)$ or equivalent equation	**2 marks for complete correct solution** **1 mark for partial correct solution**
H5,P8	**iii)** Gradient of tangent is $m_1 = \tan 45° = 1$ $\frac{dy}{dx} = 2x - 4$ hence $2x - 4 = 1$ $\therefore x = \frac{5}{2}, \quad y = \left(\frac{5}{2}\right)^2 - 4\left(\frac{5}{2}\right) + 1 = -\frac{11}{4}$ $\therefore$ The tangent is inclined at an angle of 45° to the curve at the point $\left(\frac{5}{2}, -\frac{11}{4}\right)$.	**2 marks for complete correct solution** **1 mark for partial correct solution** [Students must name both coordinates of the point]
H6,H7	**b)i)** $x < -3, x > -1$	**1 mark for complete correct solution**
H6, H7	**ii)** $x < -2$	**1 mark for complete correct solution**
H6, H7	**iii)** No, since it is not a stationary point i.e. $\frac{dy}{dx} \neq 0$ at $(-2, 2)$.	**1 mark for complete correct solution**